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Theories I. The Technifermion Sector**

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# Vacuum Alignment in Technicolor Theories

## I. The Technifermion Sector

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### Abstract

We have carried out numerical studies of vacuum alignment in technicolor models of electroweak and flavor symmetry breaking. The goal is to understand alignment's implications for strong and weak CP nonconservation in quark interactions. In this first part, we restrict our attention to the technifermion sector of simple models. We find several interesting phenomena, including (1) the possibility that all observable phases in the technifermions' unitary vacuum-alignment matrix are integer multiples of  $\pi/N'$  where  $N' \leq N$ , the number of technifermion doublets, and (2) the possibility of exceptionally light pseudoGoldstone technipions.

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# 1. Introduction

One of the original motivations for the dynamical approach to electroweak *and* flavor symmetry breaking—specifically, technicolor [1] and extended technicolor [2, 3]—was the belief that it would solve the problem of strong CP-violation in QCD [4]. The idea was this: In a theory consisting only of gauge interactions of *massless* fermions, instanton angles such as  $\theta_{QCD}$  may be freely rotated to zero. Purely dynamical masses, i.e., fermion bilinear condensates, may be assumed to be CP-conserving. And, the fermions’ hard masses are generated by the joint action of dynamical and explicit chiral symmetry breaking, all induced by gauge interactions *alone*. It was hoped that this combination naturally would produce a for which  $\bar{\theta}_q = \theta_{QCD} + \arg \det(M_q) = 0$  without an axion. This is naive, especially if at least some of the observed CP-violation is to emerge from diagonalizing the quark mass matrix  $M_q$ .

In fact, the way to determine the true status of CP symmetry in a superficially CP-invariant theory was prescribed long ago by Dashen [5]. He studied the question of determining the correct perturbative ground state  $|\Omega\rangle$  upon which to begin an expansion about the chiral limit. This process is known as vacuum alignment. When the chiral symmetry of quarks is spontaneously broken, there are infinitely many degenerate vacua, parameterized by transformations corresponding to massless Goldstone bosons. Dashen showed that, if this chiral symmetry is also explicitly broken by  $\mathcal{H}'_q = \bar{q}_L M_q q_R + \text{h.c.}$ , the degeneracy is lifted and the correctly aligned zeroth-order ground state  $|\Omega\rangle$  is the one in which the expected value of  $\mathcal{H}'_q$  is least. In practice, it is easier to fix  $|\Omega\rangle$  as a “standard vacuum” with simple condensates<sup>1</sup> and chirally rotate  $\mathcal{H}'_q$  to find the minimum vacuum energy. Dashen showed that, even if the original  $\mathcal{H}'_q$  is CP-conserving, i.e., if  $M_q$  is real, the Hamiltonian aligned with  $|\Omega\rangle$  may be CP-violating. This is spontaneous CP-violation. For real  $M_q$ , it occurs if  $\bar{\theta}_q = \pi$ . The aligned Hamiltonian has the CP-violating term  $i\nu_q \bar{q}\gamma_5 q$ , where  $\nu_q$  is of order the smallest eigenvalue of  $M_q$  [6].

Dashen’s study was made in the context of QCD, but it applies to a theory in which QCD is united with technicolor to generate quark masses by extended technicolor [7]. In such a theory, the chiral symmetries of tech-

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<sup>1</sup>We assume that the quark chiral symmetry  $SU(2n)_L \otimes SU(2n)_R$  is spontaneously broken to an  $SU(2n)$  subgroup, in which case the quark condensates  $\langle \bar{q}_{aL} q_{bR} \rangle$  are proportional to an  $SU(2n)$  matrix. In the standard vacuum,  $\langle \Omega | \bar{q}_{aL} q_{bR} | \Omega \rangle \propto \delta_{ab}$ .

nifermions are spontaneously broken at  $\Lambda_{TC} \sim 1$  TeV, giving rise to massless technipions,  $\pi_T$ . All but the three  $\pi_T^{\pm,0}$  that become the longitudinal components of the  $W^\pm$  and  $Z^0$  bosons must get large masses, at least 50–70 GeV for the charged ones. Quark chiral symmetries are spontaneously broken at the much lower scale  $\Lambda_{QCD} \sim 1$  GeV.<sup>2</sup> All these symmetries, except electroweak  $SU(2) \otimes U(1)$ , are explicitly broken by ETC–boson exchange interactions. They are well–approximated at 1 TeV by four–fermion interactions,  $\bar{T}T\bar{T}T$  and  $\bar{q}T\bar{T}q$ , suppressed by the square of  $M_{ETC} \gtrsim 100$  TeV.<sup>3</sup>

It is natural to assume that ETC breaking is such that these four–fermion interactions have real coefficients and so are superficially CP–conserving. Vacuum alignment then has three possible outcomes: (1) the correct chiral–breaking perturbation,  $\mathcal{H}'$ , is still CP–conserving and, in particular, the Cabibbo–Kobayashi–Maskawa (CKM) matrix is real; (2)  $\mathcal{H}'$  is CP–violating, but  $|\nu_q| \sim m_u$  is  $10^9$  times too large; (3)  $\mathcal{H}'$  is CP–violating, but  $|\nu_q| = 0$  or is at most is of order the ratio of condensates  $\langle \bar{q}q \rangle / \langle \bar{T}T \rangle \lesssim 10^{-9}$ . This last alternative, of course, is the desired one. Unfortunately, no physical criteria were found to lead to models of type 3.

The matter rested there until the dynamical attempts known as topcolor–assisted technicolor (TC2) were made to deal with the large mass of the top quark [8, 9]. It has always been difficult for the dynamical approach, especially extended technicolor, to account for the top quark’s mass. Either the ETC scale generating the top mass must be near 1 TeV, leading to conflict with experimental measurements on the  $\rho$  parameter [10] and the  $Z \rightarrow \bar{b}b$  decay rate [11], or, if it is made much higher, the coupling  $g_{ETC}$  must be unnaturally fine–tuned. Hill circumvented these difficulties by invoking another strong interaction near 1 TeV, topcolor, to generate a large  $\bar{t}t$  condensate and top mass. In TC2, ordinary technicolor remains responsible for the bulk of electroweak symmetry breaking.

An important consequence of this scenario—and this is where vacuum alignment comes back in—is that top condensation implies a triplet of massless Goldstone “top–pions”,  $\pi_t^{\pm,0}$ . These must acquire mass  $M_{\pi_t} \gtrsim m_t = 175$  GeV; otherwise  $t \rightarrow b\pi_t^+$  becomes a major decay mode. Extended technicolor interactions provide this mass by contributing 5–10 GeV to  $m_t$  [8]. At

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<sup>2</sup>Complications due to the top quark will be discussed below.

<sup>3</sup>To a lesser extent, the electroweak interactions also contribute to explicit symmetry breaking; see Ref. [7]. They are ignored in Eq. (2) below.

the same time, this ETC contribution must not induce appreciable mixing of top-pions with ordinary technipions [12]. Some technipions may be as light as 100 GeV [13], so that large mixing would lead to substantial, and also unobserved,  $t \rightarrow b\pi_T^+$ .

Balaji studied top-pion mass and mixing in a specific model, and he obtained encouraging results [14]. However, his conclusions are preliminary because he was unable to execute vacuum alignment properly. This is understandable because vacuum alignment in TC2 models is very complicated. Now it involves at least two gauge interactions strong near 1 TeV—technicolor and topcolor—with some technifermions transforming under both. And, many technifermions are needed to accommodate various experimental constraints, making the chiral flavor group quite large; see Ref. [9] for details. One of these experimental constraints is that no physical technipion be massless or very light. The criterion used in Refs. [9, 14] for deciding this was that no spontaneously broken chiral charge (other than the electroweak charges) can commute with the ETC-generated  $\bar{T}TTT$  interactions. We shall see in Section 3 that this criterion, which works in QCD, is *insufficient* to guarantee that all technipions are massive.

The problem of vacuum alignment in technicolor theories is too complex for analytical treatment. Numerical methods are needed. We start the numerical analysis in this paper by considering the technifermion sector of a simple ETC model, one in which there are  $N$  doublets of a single type of technifermion that transforms according to the complex fundamental representation of the technicolor gauge group  $SU(N_{TC})$ .

The rest of this paper proceeds as follows: In Section 2 we define our simplified ETC model and present the formalism in first-order chiral perturbation theory for vacuum alignment and calculating technipion masses. There we illustrate the unexpected (to us, anyway) fact that chiral symmetries are not always manifest in the chiral-breaking perturbation  $\mathcal{H}'$ . We present in Section 3 the main results of vacuum alignment in the technifermion sector. We have found a quite surprising result: the phases in the technifermions' unitary vacuum-alignment matrix  $W_0$  may be integer multiples of  $\pi/N'$  where  $N' \leq N$ . If they are allowed by unitarity, these “rational phases” occur because the terms in  $\mathcal{H}'$  make it energetically favorable for certain phases to be equal or to differ by  $\pi$  and because  $W_0$  is unimodular. If unitarity frustrates this alignment of phases in  $W_0$ , they are irrational. We shall see that the rational phases appear as islands in an irrational sea, the boundaries of

which are defined by critical values of the parameters in  $\mathcal{H}'$ . Furthermore, a technipion becomes massless, a Goldstone boson to first order, at the island shore, where the ETC parameters become critical. This has the important phenomenological consequence that an exceptionally light technipion often accompanies the rational phases because generically chosen parameters are not far from the critical ones. Thus, some technipions may be even lighter than we expected [13], a fact which may be welcome and which, in any case, can be used to help choose among models. We conclude in Section 4 with a brief look ahead to vacuum alignment and CP-violation in the quark sector.

## 2. The Extended Technicolor Model

To simplify our numerical studies, we consider models in which a single kind of technifermion interacts with quarks (but no leptons) via ETC interactions. There are  $N$  technifermion doublets  $(U_{iL,R}, D_{iL,R})$ ,  $i = 1, 2, \dots, N$ , all transforming according to the fundamental representation of the technicolor gauge group  $SU(N_{TC})$ . There are  $n$  generations of  $SU(3)_C$  triplet quarks  $(u_{aL,R}, d_{aL,R})$ ,  $a = 1, 2, \dots, n$ . The left-handed fermions are electroweak  $SU(2)$  doublets and the right-handed ones are singlets. Here and below, we exhibit only flavor, not technicolor nor color, indices. Although it is not essential for our studies, we shall assume that the technicolor gauge coupling runs slowly, or “walks” from the TC to the ETC scale [16]. No provision to give a realistic top quark mass, such as topcolor-assisted technicolor [8], will be made in this paper.

The technifermions are ordinary color-singlets, so the chiral flavor group of our model is  $G_f = [SU(2N)_L \otimes SU(2N)_R] \otimes [SU(2n)_L \otimes SU(2n)_R]$ . We have excluded anomalous  $U_A(1)$ ’s strongly broken by TC and color instanton effects. When the TC and QCD couplings reach their required critical values, these symmetries are spontaneously broken to  $S_f = SU(2N) \otimes SU(2n)$ . We shall take this residual symmetry to be the diagonal vectorial one by adopting as our standard vacuum the state  $|\Omega\rangle$  in which the nonzero fermion bilinear condensates are diagonal:

$$\begin{aligned} \langle \Omega | \bar{U}_{iL} U_{jR} | \Omega \rangle &= \langle \Omega | \bar{D}_{iL} D_{jR} | \Omega \rangle = -\delta_{ij} \Delta_T \\ \langle \Omega | \bar{u}_{aL} u_{bR} | \Omega \rangle &= \langle \Omega | \bar{d}_{aL} d_{bR} | \Omega \rangle = -\delta_{ab} \Delta_q. \end{aligned} \quad (1)$$

The condensates  $\Delta_T \simeq N_{TC} \Lambda_{TC}^3$  and  $\Delta_q \simeq N_C \Lambda_{QCD}^3$  when they are renor-

malized at their respective strong interaction scales. Of course,  $N_C = 3$ .

All of the  $G_f$  symmetries except for the gauged electroweak  $SU(2) \otimes U(1)$  must be *explicitly* broken by extended technicolor interactions [2, 3]. In the absence of a concrete ETC model, we write the interactions broken at the scale  $M_{ETC}/g_{ETC} = \mathcal{O}(100 \text{ TeV})$  in the phenomenological four-fermion form <sup>4</sup>

$$\begin{aligned} \mathcal{H}' &\equiv \mathcal{H}'_{TT} + \mathcal{H}'_{Tq} + \mathcal{H}'_{qq} \\ &= \Lambda_{ijkl}^{TT} \bar{T}_{iL} \gamma^\mu T_{jL} \bar{T}_{kR} \gamma_\mu T_{lR} + \Lambda_{iabj}^{Tq} \bar{T}_{iL} \gamma^\mu q_{aL} \bar{q}_{bR} \gamma_\mu T_{jR} + \text{h.c.} \\ &+ \Lambda_{abcd}^{qq} \bar{q}_{aL} \gamma_\mu q_{bL} \bar{q}_{cR} \gamma_\mu q_{dR}, \end{aligned} \quad (2)$$

where  $T_{iL,R}$  and  $q_{aL,R}$  stand for all  $2N$  technifermions and  $2n$  quarks, respectively. Here,  $M_{ETC}$  is a typical ETC gauge boson mass and the  $\Lambda$  coefficients are  $g_{ETC}^2/M_{ETC}^2$  times mixing factors for these bosons and group theoretical factors. Typically, the  $\Lambda$ 's are positive, though some may be negative. In our calculations, we choose the  $\Lambda$ 's to avoid unwanted Goldstone bosons. Hermiticity of  $\mathcal{H}'$  requires

$$(\Lambda_{ijkl}^{TT})^* = \Lambda_{jilk}^{TT}, \quad (\Lambda_{iabj}^{Tq})^* = \Lambda_{ajib}^{Tq}, \quad (\Lambda_{abcd}^{qq})^* = \Lambda_{badc}^{qq}. \quad (3)$$

The assumption of time-reversal invariance for this theory before any potential breaking via vacuum alignment means that the angles  $\theta_{TC} = \theta_{QCD} = 0$  (at tree level) and that all the  $\Lambda$ 's are real. Thus, e.g.,  $\Lambda_{ijkl}^{TT} = \Lambda_{jilk}^{TT}$ .

All the four-fermion operators in  $\mathcal{H}'$  are renormalized at the ETC scale. Throughout this work, we shall assume that the ETC gauge symmetries commute with electroweak  $SU(2)$ , but not with weak hypercharge  $U(1)$  (indeed, they must not; see Ref. [2]). The ETC interactions then take the form, e.g.,

$$\mathcal{H}'_{TT} = \left( \bar{U}_{iL} \gamma^\mu U_{jL} + \bar{D}_{iL} \gamma^\mu D_{jL} \right) \left( \Lambda_{ijkl}^U \bar{U}_{kR} \gamma_\mu U_{lR} + \Lambda_{ijkl}^D \bar{D}_{kR} \gamma_\mu D_{lR} \right). \quad (4)$$

Having chosen a standard chiral-perturbative ground state,  $|\Omega\rangle$ , vacuum alignment proceeds by minimizing the expectation value of the rotated Hamiltonian. This is obtained by making the  $G_f$  transformation  $T_{L,R} \rightarrow W_{L,R} T_{L,R}$  and  $q_{L,R} \rightarrow V_{L,R} q_{L,R}$ , where  $W_{L,R} \in SU(2N)_{L,R}$  and  $V_{L,R} \in$

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<sup>4</sup>In Eq. (2), we have not made any assumption about the structure of ETC interactions vis-a-vis the electroweak ones.



$SU(2n)_{L,R}$ :

$$\begin{aligned}\mathcal{H}'(W, V) &= \mathcal{H}'_{TT}(W_L, W_R) + \mathcal{H}'_{Tq}(W, V) + \mathcal{H}'_{qq}(V_L, V_R) \\ &= \Lambda_{ijkl}^{TT} \bar{T}_{i'L} W_L^\dagger \gamma^\mu W_{Ljj'} T_{j'L} \bar{T}_{k'R} W_R^\dagger \gamma^\mu W_{Rll'} T_{l'L} + \dots\end{aligned}\quad (5)$$

Since  $T$  and  $q$  transform according to complex representations of their respective color groups, the four-fermion condensates in the  $S_f$ -invariant  $|\Omega\rangle$  have the form

$$\begin{aligned}\langle \Omega | \bar{T}_{iL} \gamma^\mu T_{jL} \bar{T}_{kR} \gamma_\mu T_{lR} | \Omega \rangle &= -\Delta_{TT} \delta_{il} \delta_{jk}, \\ \langle \Omega | \bar{T}_{iL} \gamma^\mu q_{aL} \bar{q}_{bR} \gamma_\mu T_{jR} | \Omega \rangle &= -\Delta_{Tq} \delta_{ij} \delta_{ab}, \\ \langle \Omega | \bar{q}_{aL} \gamma^\mu q_{bL} \bar{q}_{cR} \gamma_\mu q_{dR} | \Omega \rangle &= -\Delta_{qq} \delta_{ad} \delta_{bc}.\end{aligned}\quad (6)$$

The condensates are positive, renormalized at  $M_{ETC}$  and, in the large- $N_{TC}$  and  $N_C$  limits, they are given by  $\Delta_{TT} \simeq (\Delta_T(M_{ETC}))^2$ ,  $\Delta_{Tq} \simeq \Delta_T(M_{ETC}) \times \Delta_q(M_{ETC})$ , and  $\Delta_{qq} \simeq (\Delta_q(M_{ETC}))^2$ . In walking technicolor,  $\Delta_T(M_{ETC}) \simeq (M_{ETC}/\Lambda_{TC}) \Delta_T(\Lambda_{TC}) = 10^2\text{--}10^3 \times \Delta_T(\Lambda_{TC})$ . In QCD, however,  $\Delta_q(M_{ETC}) \simeq (\log(M_{ETC}/\Lambda_{QCD}))^{\gamma_m} \Delta_q(\Lambda_{ETC}) \simeq \Delta_q(\Lambda_{QCD})$ , where  $\gamma_m \simeq 2\alpha_C/\pi$  for  $SU(3)_C$  [17]. Thus, the ratio

$$r = \frac{\Delta_{Tq}(M_{ETC})}{\Delta_{TT}(M_{ETC})} \simeq \frac{\Delta_{qq}(M_{ETC})}{\Delta_{Tq}(M_{ETC})} \quad (7)$$

is at most  $10^{-10}$ . This is 10–100 times smaller than in a technicolor theory in which the coupling does not walk.

With these condensates, the vacuum energy is a function only of  $W = W_L W_R^\dagger$  and  $V = V_L V_R^\dagger$ , elements of the coset space  $G_f/S_f$ :

$$\begin{aligned}E(W, V) &= E_{TT}(W) + E_{Tq}(W, V) + E_{qq}(V) \\ &= -\Lambda_{ijkl}^{TT} W_{jk} W_{li}^\dagger \Delta_{TT} - \left( \Lambda_{iabj}^{Tq} V_{ab} W_{ji}^\dagger + \text{c.c.} \right) \Delta_{Tq} - \Lambda_{abcd}^{qq} V_{bc} V_{da}^\dagger \Delta_{qq}.\end{aligned}\quad (8)$$

Note that time-reversal invariance of the unrotated Hamiltonian  $\mathcal{H}'$  implies that  $E(W, V) = E(W^*, V^*)$ . Hence, spontaneous CP-violation occurs if the solutions  $W_0, V_0$  to the minimization problem are complex.

Following Ref. [7], we define technifermion current mass matrices renormalized at the ETC scale as follows:<sup>5</sup>

$$M_{Tij} \Delta_T(M_{ETC}) = - \left( W_{ik}^\dagger \frac{\partial E}{\partial W_{jk}^\dagger} \right)_{W_0, V_0}$$

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<sup>5</sup>These definitions differ from those in Ref. [7] by a common vectorial transformation on the left and right-handed fields. This affects none of our discussion.

$$\begin{aligned}
&= W_{0ik}^\dagger \left( \Lambda_{klmj}^{TT} W_{0lm} \Delta_{TT} + \Lambda_{kabj}^{Tq} V_{0ab} \Delta_{Tq} \right) \\
&= W_{0ik}^\dagger \Lambda_{klmj}^{TT} W_{0lm} \Delta_{TT} (1 + \mathcal{O}(r)) .
\end{aligned} \tag{9}$$

For quarks,

$$\begin{aligned}
M_{qab} \Delta_q(M_{ETC}) &= - \left( V_{ac}^\dagger \frac{\partial E}{\partial V_{bc}^\dagger} \right)_{W_0, V_0} \\
&= V_{0ac}^\dagger \left( \Lambda_{cijb}^{Tq} W_{0ij} \Delta_{Tq} + \Lambda_{cdeb}^{qq} V_{0de} \Delta_{qq} \right) \\
&= V_{0ac}^\dagger \Lambda_{cijb}^{Tq} W_{0ij} \Delta_{Tq} (1 + \mathcal{O}(r)) .
\end{aligned} \tag{10}$$

Nuyts' theorem generalized to technicolor [6, 7] states that, as a consequence of extremizing the energy, the imaginary parts of these matrices are proportional to the identity:

$$\begin{aligned}
(M_T - M_T^\dagger) \Delta_T(M_{ETC}) &= i\nu_T 1_{2N} , \\
(M_q - M_q^\dagger) \Delta_q(M_{ETC}) &= i\nu_q 1_{2n} .
\end{aligned} \tag{11}$$

The parameters  $\nu_T$  and  $\nu_q$  are Lagrange multipliers associated with the unimodularity constraints on  $W_0$  and  $V_0$ , respectively. These equations imply that  $M_T$  and  $M_q$  are each diagonalized by a (different) single special unitary transformations. Taking the trace of both sides of Eqs. (11) and using Eqs. (9,10) gives

$$\begin{aligned}
2iN\nu_T &\equiv \text{Tr} (M_T - M_T^\dagger) \Delta_T(M_{ETC}) \\
&= -\text{Tr} (M_q - M_q^\dagger) \Delta_q(M_{ETC}) \equiv -2in\nu_q \\
&= 2i\Lambda_{kabj}^{Tq} \text{Im} (W_{0kj}^* V_{0ab}) \Delta_{Tq} .
\end{aligned} \tag{12}$$

This relation between  $\nu_T$  and  $\nu_q$  requires that  $SU(N_{TC})$  and  $SU(3)_C$  are embedded in a simple ETC group, so that  $\theta_{TC} = \theta_{QCD}$ .

Strong CP-violation occurs if  $\nu_{T,q} \neq 0$ . The angle  $\bar{\theta}_q$  characterizing this for quarks' is defined by (for  $\theta_{QCD} = 0$ ) <sup>6</sup>

$$\bar{\theta}_q = \arg \det (M_q) \Delta_q = \arg \det (\mathcal{M}_q) \Delta_q , \tag{13}$$

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<sup>6</sup>See, e.g., Ref. [19] for the relation between  $\theta_q$  and  $\nu_q$  for the case of three light quarks.

where, up to  $\Lambda^{qq}$  terms of relative order  $r$ ,

$$\mathcal{M}_{qab} \Delta_q \equiv (V_0 M_q)_{ab} \Delta_q = \Lambda_{aijb}^{T_q} W_{0ij} \Delta_{T_q} \quad (14)$$

is the primordial quark mass matrix, i.e., the one *before* vacuum alignment in the quark sector. We see that strong CP-violation arises from a conflict between mass terms and a chiral symmetry constraint on the alignment matrix. This is what Dashen and Nuyts showed for quarks in QCD and what we found in Ref. [7] for extended technicolor. In a world with just one type of fermion, say  $T_{iL,R}$ , with explicit flavor symmetry breaking due to gauge interactions *alone*,  $M_T = M_T^\dagger$  and there is no strong CP-violation even if the aligning matrix  $W_0$  is complex and CP symmetry is spontaneously broken,

Suppose we found  $\bar{\theta}_q = 0$  up to the  $\Lambda^{qq}$  terms of order  $r$ . Are there larger contributions to  $\bar{\theta}_q$ ? The first to worry about are two-loop ETC contributions to  $\mathcal{M}_q$ . There are two types of these: those with one technifermion dynamical mass insertion and those with three. The first are proportional to a single power of  $W_0$  and, because the  $\Lambda^{TT}$ 's are real, it is plausible that they will not change  $\bar{\theta}_q$ . This must be checked in specific models. The three-insertion graphs involve two  $W_0$ 's and one  $W_0^\dagger$  convoluted with  $\Lambda^{TT}$ 's and these are more likely to contribute to  $\bar{\theta}_q$ . Apart from any  $g_{ETC}^2/16\pi^2$  suppression these graphs may have, they are of relative order  $\Delta_T^2(M_{ETC})/M_{ETC}^6 \lesssim (\Lambda_{TC}/M_{ETC})^4 \lesssim 10^{-10}$  in a walking technicolor theory. We tentatively conclude that the  $\bar{\theta}_q$  defined in Eq. (13) is a reliable measure of strong CP-violation in extended technicolor models. We need only know  $W_0$  and the  $\Lambda^{T_q}$  to determine it.

Our strategy for vacuum alignment, which we carry out numerically, is the following: Because  $r$  is small, we first minimize  $E_{TT}$  to determine  $W_0$ . If we wish to determine  $W_{0L}$  and  $W_{0R}$  separately, we make vectorial transformations on  $T_{L,R}$  that diagonalize  $M_{Tij}$ . Physical results such as technipion and quark masses are unchanged even if we use, for example,  $W_{0L} = W_0$ . The results of technifermion alignment are presented in the next section.

Once  $W_0$  is determined, it is inserted as a set of parameters into  $E_{Tq}$  and this is minimized as a function of  $V$ . If there are several degenerate solutions  $W_0$  minimizing  $E_{TT}$ , one should choose the one giving the deepest minimum  $E_{Tq}(W_0, V_0)$ . When  $V_0$  is known, the matrices  $V_{0L}$ ,  $V_{0R}$  are determined by diagonalizing the matrix  $M_q$  in Eq. (10). The quark CKM matrix is then obtained from  $V_{0L}$ .

Finally, holding  $V_0$  fixed, one can refine  $W_0$  by reminimizing  $E_{TT} + E_{Tq}$  as a function of  $W$ . This will induce corrections of  $\mathcal{O}(r)$  in  $W_0$  and  $\bar{\theta}_q$ . There is no point in refining  $V_0$  by minimizing the full energy including  $E_{qq}$ . However, note that the rotated  $\mathcal{H}'_{qq}(V_0)$  may contain sources of quark CP-violation not contained in the CKM matrix [7]. These studies of CP-violation in the quark sector will be presented in our next paper.

We are concerned in this paper with vacuum alignment in the technifermion sector, and we turn to this now. We will allow only models in which alignment conserves electric charge, i.e., does not induce  $\bar{U}_i D_j$  condensates. Then, the matrix minimizing  $E_{TT}(W)$  must be block-diagonal,

$$W_0 = \begin{pmatrix} W_0^U & 0 \\ 0 & W_0^D \end{pmatrix}, \quad (15)$$

where  $W_0^U, W_0^D$  are  $U(N)$  matrices satisfying  $\det(W_0^U) \det(W_0^D) = 1$ . The phase indeterminacy of the individual  $U(N)$  matrices corresponds to the electroweak  $T_3$  symmetry. Thus, for admissible models, we can minimize  $E_{TT}$  in the subspace of block-diagonal matrices. Using Eq. (4), the vacuum energy takes the form

$$\begin{aligned} E_{TT}(W^U, W^D) &= -(\Lambda_{ijkl}^U W_{jk}^U W_{li}^{U\dagger} + \Lambda_{ijkl}^D W_{jk}^D W_{li}^{D\dagger}) \Delta_{TT} \\ &\equiv E_U(W^U) + E_D(W^D). \end{aligned} \quad (16)$$

Since this expression is bilinear in  $W^{U,D} W^{U,D\dagger}$ , without loss of generality we can determine  $W_0$  by separately minimizing  $E_U$  and  $E_D$  in the space of  $SU(N)$  matrices. We do this in the next section, taking care to ensure that no Goldstone bosons remain massless other than the three associated with electroweak  $SU(2)$  symmetry. This means that  $\Lambda_{ijkl}^{U,D}$  must be chosen so that there are no massless  $SU(N) \otimes SU(N)$  Goldstone boson in either  $U$  or  $D$  sector.

We close this section with some remarks on calculating the pseudoGoldstone boson (technipion) mass spectrum in these technicolor models. In the standard chiral-perturbative ground state,  $|\Omega\rangle$ , the spontaneously broken symmetries are formally generated by the  $G_f/S_f$  charges

$$Q_5^A = \frac{1}{2} \int d^3x \left( T_R^\dagger \lambda_A T_R - T_L^\dagger \lambda_A T_L \right), \quad (17)$$

Here,  $\lambda_A$  are the  $4N^2 - 1$  Gell-Mann matrices of  $SU(2N)$ . To first order in the chiral perturbation  $\mathcal{H}'_{TT}$ , technipion masses are given by Dashen's formula [5],

$$F_T^2 M_{\pi AB}^2 = i^2 \left\langle \Omega \left| \left[ Q_5^A, \left[ Q_5^B, \mathcal{H}'_{TT}(W_{0L}, W_{0R}) \right] \right] \right| \Omega \right\rangle, \quad (18)$$

where  $F_T = 246 \text{ GeV}/\sqrt{N}$  is the technipion decay constant and  $\mathcal{H}'_{TT}$  is given in Eq. (5). As noted above, we can determine  $W_{0L}$  and  $W_{0R}$  by diagonalizing the technifermion current-mass matrix,  $M_T$ . However, since  $M_{\pi AB}^2$  is invariant under vectorial transformations of the technifermion fields, it is simpler to compute it using  $W_{0L} = W_0$  and  $W_{0R} = 1$ . The result is

$$\begin{aligned} F_T^2 M_{\pi AB}^2 = \frac{1}{2} \Lambda_{ijkl} & \left[ \left( \{ \lambda_A, \lambda_B \} W_0^\dagger \right)_{li} W_{0jk} + (W_0 \{ \lambda_A, \lambda_B \})_{jk} W_{0li}^\dagger \right. \\ & \left. - 2 \left( \lambda_A W_0^\dagger \right)_{li} (W_0 \lambda_B)_{jk} - 2 \left( \lambda_B W_0^\dagger \right)_{li} (W_0 \lambda_A)_{jk} \right] \Delta_{TT}. \end{aligned} \quad (19)$$

Note that, because the vector charges annihilate the standard vacuum, the axial charges in Eq. (19) may be replaced by left-handed or right-handed charges or by any linear combination that is not purely a vector charge.

In using Eq. (19) in these models, we have seen examples in which a technipion's mass vanishes without there being a corresponding conserved chiral charge, i.e., a linear combination of the  $Q_R^A$  and  $Q_L^A$  which commutes with  $\mathcal{H}'_{TT}(W_0)$ . A two-technidoublet,  $SU(4) \otimes SU(4)$  example is provided by the following set of  $\Lambda$ 's (whose scale is arbitrary):

$$\begin{aligned} \Lambda_{1111}^U &= \Lambda_{1111}^D = \Lambda_{2222}^U = \Lambda_{2222}^D = 1 \\ \Lambda_{1112}^U &= \Lambda_{1121}^U = \Lambda_{1112}^D = \Lambda_{2221}^D = \frac{1}{2} \\ \Lambda_{1211}^U &= \Lambda_{2111}^U = \frac{1}{2}. \end{aligned} \quad (20)$$

In addition to the three electroweak Goldstone bosons coupling to

$$\frac{1}{2} \int d^3x \sum_{i=1}^2 (U_i^\dagger, D_i) \gamma_5 \tau_a \begin{pmatrix} U_i \\ D_i \end{pmatrix},$$

there is a fourth one associated with the  $W_0$ -rotation of the axial charge

$$\frac{1}{2} \int d^3x (D_1^\dagger \gamma_5 D_1 - D_2^\dagger \gamma_5 D_2).$$

However, the divergence of its current is manifestly of first order in  $\mathcal{H}'_{TT}(W)$ .

This extra massless technipion is at first surprising when one recalls that Dashen proved that a zero eigenvalue of the Goldstone boson mass-squared matrix implies that the corresponding current is conserved [5]. Furthermore, in QCD we have become used to a conserved current being associated with a massless Goldstone boson. There, the symmetry that leaves the boson massless is manifest in the mass matrix  $M_q$  of  $\mathcal{H}'_q = \bar{q}_L M_q q_R + \text{h.c.}$ . The resolution of this puzzle is that Dashen's proof applies to the matrix elements of double commutators in the *exact* ground state,  $|\text{vac}\rangle$ , of the *full* Hamiltonian  $H = \int(\mathcal{H}_0 + \mathcal{H}'(W))$ . The matrix in Eq. (19) is calculated in the perturbative ground state,  $|\Omega\rangle$ , which is the limit of  $|\text{vac}\rangle$  as  $\mathcal{H}'(W) \rightarrow 0$ . Consequently, all that can be proved for a “massless” Goldstone boson at the perturbative level at which we work is that *all* matrix elements of the divergence of the corresponding current must be of *second* order in  $\mathcal{H}'$ . We emphasize that, although the masslessness of this technipion may be an approximation, it is important phenomenologically. Corrections to its mass are likely to be so small that it is already ruled out experimentally.

### 3. Results From the Technifermion Sector

The vacuum energy in the  $U$  and  $D$ -technifermion sectors has the form

$$\begin{aligned} E(W) &= - \sum_{ijkl=1}^N \Lambda_{ijkl} W_{jk} W_{li}^\dagger \Delta_{TT} \\ &\equiv - \sum_{ijkl=1}^N \Lambda_{ijkl} |W_{jk}| |W_{il}| \exp[i(\phi_{jk} - \phi_{il})] \Delta_{TT}, \end{aligned} \quad (21)$$

where  $W = W_U$  or  $W_D \in SU(N)$ ,  $\Lambda_{ijkl} = \Lambda_{ijkl}^* = \Lambda_{jilk}$ , and  $\phi_{jk} = \arg(W_{jk})$ . We remind the reader that we always choose the  $\Lambda_{ijkl}$  so that there is no  $SU(N) \otimes SU(N)$  Goldstone boson in either  $U$  or  $D$  sector. Note that, if  $W_0$  minimizes  $E(W)$ , then so do the matrices  $Z_N^{2m} W_0 = \exp(2im\pi/N) W_0$ ,  $m = 1, 2, \dots, N$ , and their complex conjugates. This degeneracy may be lifted by the quark-technifermion interaction  $\mathcal{H}'_{Tq}$ .

It is especially convenient to parameterize  $W$  in the form

$$W = D_L K D_R. \quad (22)$$

Here,  $D_{L,R}$  are diagonal unimodular matrices, each depending on  $N - 1$  phases:

$$D_{L,R} = \text{diag} [\exp(i\chi_{L,R1}), \exp(i\chi_{L,R2}), \dots, \exp(-i(\chi_{L,R1} + \dots + \chi_{L,RN-1}))], \quad (23)$$

and  $K$  is an  $(N - 1)^2$ -parameter CKM matrix which we write in the standard Harari–Leurer form [18]. This matrix depends on  $\frac{1}{2}N(N - 1)$  angles  $\theta_{ij}$ ,  $1 \leq i < j \leq N$ , and  $\frac{1}{2}(N - 1)(N - 2)$  phases  $\delta_{ij}$ ,  $1 \leq i < j - 1 \leq N - 1$ .

We have discovered several remarkable properties of the matrices  $W_0$  which minimize  $E(W)$ . They have to do with the fact that the coefficient  $\Lambda_{ijkl}$  tends to align the phases  $\phi_{jk}$  and  $\phi_{il}$ : if  $\Lambda_{ijkl} > 0$ , its contribution to  $E(W)$  is minimized if the phases can be equal; if  $\Lambda_{ijkl} < 0$ , the phases want to differ by  $\pi$ . Of course, because not all the  $N^2$  phases  $\phi_{jk}$  are independent, unitarity can frustrate phase alignment. If the nonzero  $\Lambda_{ijkl}$  link *all* the  $\phi_{jk}$  together, then all of them will be equal, mod  $\pi$ , *if that can be consistent with unitarity*. Unimodularity then requires all  $\phi_{jk} = 2m\pi/N$ , mod  $\pi$ , with  $m = 1, 2, \dots, N$ . We call this “complete phase alignment” and we say that the phases are “rational”.

Rational phases may also occur when the nonzero  $\Lambda_{ijkl}$  link some, but not all, of the phases. If it is allowed by unitarity, we have found that the phases are multiples of  $\pi/N'$  (modulo the  $Z_N$  phase  $2m\pi/N$ ) for one or more values of  $N'$  between 1 and  $N$ . This case of partial phase alignment is very rich, with many possibilities and, sometimes, degenerate minima whose  $W_0$ 's are not unitarily equivalent nor related by conjugation or a  $Z_N$  factor. Its implications for quark CP-violation will be studied in our next paper.

A necessary condition for phase alignment is that the CKM matrix  $K$  is real. The reason for this is seen by looking at a typical complex term in  $K$  for the  $3 \times 3$  case, e.g.,  $s_{12}s_{23} - c_{12}c_{23}s_{13}\exp(i\delta_{13})$ , where  $s_{12} = \sin\theta_{12}$ , etc. The mixing angles  $\theta_{ij}$  are determined by the  $\Lambda$ 's that are dominant in minimizing the energy and by unitarity. Then, the overall phase of this term will be a random irrational number unless  $\delta_{13} = 0$  or  $\pi$  or one of the  $\theta_{ij} = 0$ . If  $K$  is complex, it contains more random phases than can be made rational by choices of phases in  $D_L$  and  $D_R$ , and so the  $\phi_{jk}$  will be randomly irrational. Note that the case  $N = 2$  is special because  $K$  is always real. In that case, all phases in  $W_0$  are 0 or  $\pi/2$ , mod  $\pi$ .

Suppose that completely or partially-aligned rational phases occur for some set of  $\Lambda$ 's. Then we find that the nonzero  $\Lambda$ 's may be varied over an

appreciable range with no change whatever in the phases. Ultimately, a large enough excursion in the  $\Lambda$ 's will make it impossible to maintain unitarity with aligned phases and, at certain critical values of the  $\Lambda$ 's, they change continuously from rational to irrational (or, in the  $SU(2)$  case, discontinuously from one rational set to another). A rational-to-irrational phase transition may also occur if vanishing  $\Lambda$ 's are made nonzero. By further varying the  $\Lambda$ 's, another, possibly inequivalent, set of rational phases may characterize the matrix  $W_0$ . Thus, the minima of  $E(W)$  as one varies the  $\Lambda$ 's are islands of rational aligned phases in a sea of irrational ones.

A Goldstone boson appears whenever a transition occurs between different types of phases. As the critical  $\Lambda$ 's are approached, one of the  $M_\pi^2$  decreases to zero and then increases again once the boundary is passed. What is happening is this: As the transition is approached, the ground states for a set of rational phases are becoming less stable and a technipion's  $M_\pi^2$  is diving through zero to negative values. At the same time, the ground states for a nearby set of irrational phases are becoming more stable and the corresponding  $M_\pi^2$  is increasing from negative to positive values. The two types of phases coexist at the rational island shore, giving rise to infinitely many degenerate minima that are characterized by an indeterminacy in the phases of  $D_{L,R}$  and  $K$ . Hence,  $M_\pi^2 = 0$  (to first order) there. This is another situation in which the massless state's chiral charge does not commute with  $\mathcal{H}'_{TT}$ .

This phenomenon may be important. The appearance of an exceptionally light technipion is not uncommon because typical rational-phase  $\Lambda$ -parameters often are not far from critical ones. In Ref. [13] we observed that, because the number of technidoublets in typical TC2 models is large,  $N \sim 10$ , the technihadron scale is low and technipion masses may be as light as 100 GeV. Now we see that some technipions may be even lighter than nominally expected from the  $\Lambda$ 's. In a specific model, this may be a major prediction or it may be a show-stopper.

Finally, another interesting property of the rational-phase minima is that the coefficients  $\tilde{\Lambda}_{ijkl} = \sum_{i'j'} \Lambda_{i'j'kl} W_{0i'i'}^\dagger W_{0j'j}$  in the rotated Hamiltonian

$$\mathcal{H}'_{TT}(W_0) = \tilde{\Lambda}_{ijkl} \bar{T}_{Li} \gamma^\mu T_{Lj} \bar{T}_{Rk} \gamma_\mu T_{Rl}, \quad (24)$$

also have rational phases. This follows directly from the fact that nonzero  $\Lambda$ 's align phases. If the phases are rational and  $\Lambda_{i'j'kl} \neq 0$ , then the CKM matrix  $K$  is real and  $\phi_{j'k} - \phi_{i'l} = \chi_{Lj'} - \chi_{Rk} - \chi_{Li'} + \chi_{Rl} = 0 \pmod{\pi}$ .



The phase of an individual term in the sum for  $\tilde{\Lambda}_{ijkl}$  is then  $\phi_{j'j} - \phi_{i'i} = \chi_{Lj'} - \chi_{Rj} - \chi_{Li'} + \chi_{Ri} = \chi_{Rk} - \chi_{Rj} - \chi_{Ri} + \chi_{Ri} \pmod{\pi}$ , a rational phase which is the same for all terms in the sum over  $i', j'$ .

One example of these phenomena is provided by an  $SU(3)$  model in which the nonzero  $\Lambda$ 's are:

$$\begin{aligned}\Lambda_{1111} &= \Lambda_{1221} = \Lambda_{2112} = \Lambda_{1212} = \Lambda_{2121} = 1.0 \\ \Lambda_{1122} &= 1.5, \quad \Lambda_{1133} = 1.4 \\ \Lambda_{1331} &= \Lambda_{3113} = 1.6, \quad \Lambda_{1313} = \Lambda_{3131} = 1.8 \\ \Lambda_{1222} &= \Lambda_{2122} = \Lambda_{2212} = \Lambda_{2221} = 0.50 - 1.1.\end{aligned}\tag{25}$$

These want to align  $\phi_{11} = \phi_{22} = \phi_{33} = \phi_{12} = \phi_{21}$  and  $\phi_{13} = \phi_{31}$ . Phases  $\phi_{23}$  and  $\phi_{32}$  are not linked by the  $\Lambda$ 's. The effect of varying  $\Lambda_{1222}$  from 0.5 to 1.1 is illustrated in Fig. 1. The phases start out aligned and rational, indeed,  $W = \exp(2i\pi/3) \times 1$ , and the vacuum energy (in units of  $\Delta_{TT}$ ) remains constant at  $-6.20$ . At  $\Lambda_{1222} \simeq 0.725$ , it becomes energetically favorable for  $W$  to become nondiagonal. The phases are still aligned and rational, equal to  $2\pi/3 \pmod{\pi/2}$ , and a technipion mass becomes zero here. Now the energy decreases as  $\Lambda_{1222}$  is increased. At  $\Lambda_{1222} \simeq 1.015$ , a second transition occurs in which rational phases are no longer possible, and a different technipion's mass goes through zero. At  $\Lambda_{1222} \simeq 1.045$ , a transition occurs back to rational phases, all equal to  $\pi/3 \pmod{\pi}$ , and the same technipion's mass vanishes again. Throughout this variation of  $\Lambda_{1222}$ , the other six technipion squared masses remain fairly constant with values between 5 and 15. Thus, in this example, the two technipion masses shown in Fig. 1 are always quite light.

## 4. Summary and Outlook

We have numerically studied vacuum alignment in a class of theories in which electroweak and flavor symmetries are dynamically broken by gauge interactions alone. To make these initial studies tractable, we considered extended technicolor with  $N$  doublets of a single type of technifermion,  $T_{iL,R}$ , transforming according to a complex representation of  $SU(N_{TC})$  but as  $SU(3)_C$  singlets. These were coupled by ETC to  $n$  quark doublets,  $q_{aL,R}$ . In the absence of an explicit model for ETC, we assumed its broken gauge interactions could produce any desired four-fermion interaction of the form  $\mathcal{H}'$  in Eq. (2).

As usual, we assumed that ETC commutes with electroweak  $SU(2)$ , but not  $SU(N_{TC}) \otimes SU(3)_C \otimes U(1)$  [2]. We also assumed, quite naturally, that ETC breaking preserves CP-invariance so that the  $\Lambda$  parameters in  $\mathcal{H}'$  are real.

We focussed on the technifermion sector in this paper. This restriction determines the vacuum-aligning matrix  $W_0$  of technifermions up to tiny, but potentially important corrections of order  $\langle \bar{q}q \rangle_{ETC} / \langle \bar{T}T \rangle_{ETC} \sim 10^{-10}$ . The problem is then simplified both numerically and analytically to minimizing the vacuum energy  $E_{TT}(W)$  in the subspace of up and down-block diagonal  $W$ -matrices which conserve electric charge. We need then only study the alignment problem in a single charge sector. To ensure that no technipions remain massless other than the three associated with electroweak symmetry, the ETC parameters  $\Lambda_{ijkl}^{U,D}$  in  $\mathcal{H}'_{TT}$  must be chosen so that there are no  $SU(N) \otimes SU(N)$  Goldstone boson in either  $U$  or  $D$  sector.

We found several interesting features of vacuum alignment:

1. A technipion mass may vanish to first order in the symmetry breaking perturbation even if its chiral charge does not commute with  $\mathcal{H}'_{TT}(W_0)$ . This differs from what happens in QCD and  $\Sigma$ -model-like effective Lagrangians where the symmetries of the perturbation are manifest. The reason for this difference is the four-fermion nature of  $\mathcal{H}'_{TT}$  and the symmetries of the zeroth-order ground state  $|\Omega\rangle$ .
2. The real parameter  $\Lambda_{ijkl}^{U,D}$  links the  $W^{U,D}$  phases  $\phi_{jk}^{U,D}$  and  $\phi_{il}^{U,D}$ . If allowed by unitarity of  $W$ , these phases are then equal or differ by  $\pi$ . If there is complete phase alignment, all phases are equal to integer multiples of  $2\pi/N$ , mod  $\pi$ . If only partially aligned, the phases are integer multiples of  $\pi/N'$  for one or more  $N' \leq N$ . If phase alignment is inconsistent with unitarity, the phases are irrational multiples of  $\pi$ .
3. Rational phase sets are natural in the sense that they remain unchanged for a finite range of  $\Lambda$  parameters. In  $\Lambda$ -space, the rational phase solutions to vacuum alignment form discrete islands in a sea of irrational phase solutions.
4. A massless (to first order) Goldstone boson appears when the  $\Lambda$ 's take on critical values defining the boundary between rational and irrational phases. Thus, exceptionally light technipions are not at all uncommon and are a new phenomenological consequence of vacuum alignment.

Vacuum alignment in the quark sector and the central issue of quark CP-violation will be addressed in a subsequent paper. It is obvious from Eq. (12) that irrational phases in the technifermion matrix  $W_0$  will induce strong CP-violation for quarks:  $\nu_q \neq 0$ . It is therefore fortunate that rational phases occur naturally. They may permit a dynamical theory of quark flavor in which only weak CP-violation occurs and in which there is no axion.

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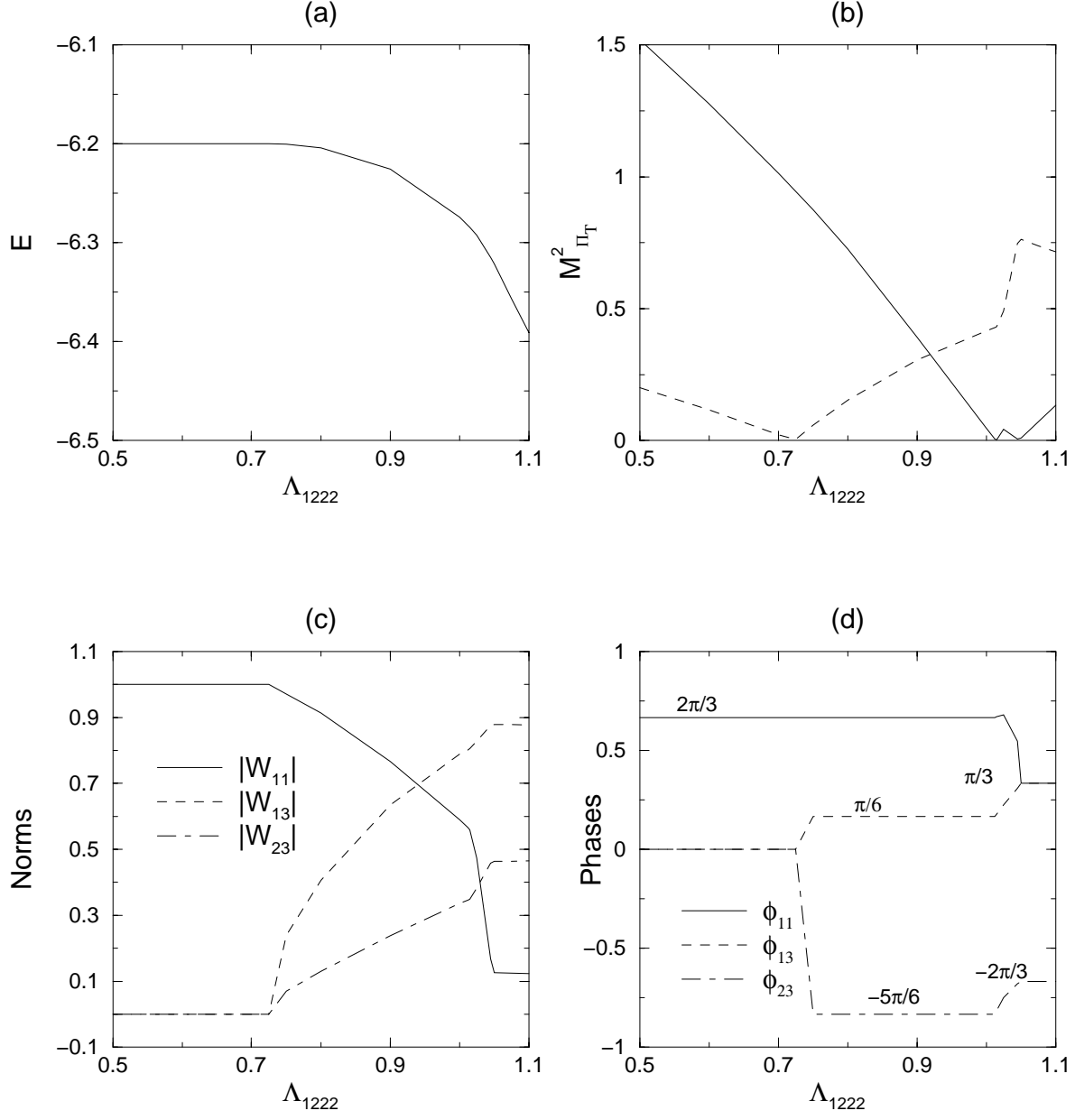


Figure 1: Phase alignment in a model with  $N = 3$  as a function  $\Lambda_{1222}$ ; other  $\Lambda$ -parameters are fixed in Eq. (25). (a) The vacuum energy,  $E(W)$  (arbitrary units); (b) the squared mass of two of the technipions; (c), (d) the magnitudes and phases of  $W_{11}$ ,  $W_{13}$  and  $W_{23}$ .